From Possibility to Responsibility

A Deduction in Homotopy Type Theory from a Single Axiom
to the Coherence Conditions of Adequate Self-Reference

Dr. Markus Killius

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Abstract

This paper presents a deduction within Homotopy Type Theory (HoTT) that proceeds from a single axiom – *there is possibility* – through fourteen steps to a formally derived coherence condition: any structurally adequate self-referential type in the presence of irreducible alterity has exactly one non-defective mode of operation – relation without reduction. This result is philosophically interpreted as the formal structure of responsibility. The deduction runs in two parallel layers: a formal layer consisting of type-theoretic constructions, definitions, and derivations, and a philosophical layer providing interpretation of each formal structure. These layers are strictly separated. The point at which formal structure necessitates interpretive supplementation – the transition from ontology to epistemology – is explicitly marked. Schematic notation is identified as such. No additional axiom beyond the foundational one (and the standard framework of HoTT including the Univalence Axiom) is introduced. The strength of the concluding result depends on a definitional choice – the adequacy condition for self-reference – whose justification via Univalence is given and whose status as a definitional choice is made transparent.

Contents

1 Prolegomena

1.1 On the Choice of Formal System

Homotopy Type Theory is selected as the formal framework for structural reasons, not by convention. The choice is motivated by the following correspondences:

- **Identity as equivalence.** The Univalence Axiom $(A =_{\mathcal{U}} B) \simeq (A \simeq B)$ formally encodes that identity between structures *is* structural equivalence – not an external relation imposed upon them.
- **Native reflexivity.** The universe hierarchy $\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$ provides structure over structure without circularity.
- **Proof as inhabitant.** The Curry-Howard correspondence (propositions as types, proofs as terms) ensures that every claim is a construction, not merely an assertion.
- **Dimensional richness.** The path structure (identity types, homotopies, higher homotopies) provides a native formal language for layered structure.

Transparency Note

The choice of HoTT is not neutral. It is consonant with the axiom: the Univalence Axiom formally expresses what "there is possibility" philosophically posits – that identity is structural equivalence. This consonance is a reason for the choice, not a circularity. The formal deduction depends on HoTT's standard machinery; the axiom **A0** is the sole addition.

1.2 On the Meta-Theoretic Status of HoTT

A crucial distinction must be made explicit at the outset. The deduction presented in this paper is formalized *in* HoTT. Whether its inferential core is framework-invariant — whether the same deduction could be carried out in any sufficiently expressive formal system — is a well-motivated conjecture but not here established. It remains an open meta-theoretic claim (see Open Questions, Section 10).

What can be said with precision is this: HoTT is *sufficient*. It provides a formal language in which every step of the deduction can be rigorously expressed. The deduction relies at critical points (Steps 3 and 12) on the principle that identity between structures *is* structural equivalence — the Univalence Axiom. Any alternative formal system would need to contain a functional equivalent of this principle. Whether such equivalents exist in other frameworks (e.g., category theory, higher topos theory) is plausible but unproven.

The fact that HoTT is particularly well-suited (its native universe hierarchy, its path structure, its Univalence Axiom mirror the deduced structures with minimal friction) is not a circularity but a *sufficiency criterion*: HoTT is chosen because its structures can express the deduced structures without distortion. The consonance between framework and content is the mark of sufficiency, not of dependence.

The philosophical motivation for the conjecture of framework-invariance is this: the deduction proceeds from the axiom "there is possibility," which is logically prior to any particular formal system. If the structures derived from this axiom were artifacts of HoTT rather than consequences of the axiom, one would expect them to disappear in alternative formalizations. The conjecture is that they would not — but demonstrating this requires exhibiting the deduction in at least one alternative framework, which is deferred to future work.

1.3 Method: Two Layers

Every step below is presented in two registers:

Formal (HoTT)

Contains only type-theoretic definitions, constructions, and derivations. Notation follows the HoTT Book (Univalent Foundations Program, 2013). No philosophical terms appear here.

Philosophical Interpretation

Contains the philosophical interpretation of the formal structure. This layer assigns conceptual names ("difference," "relation," "consciousness," "responsibility") to the formal constructs. The assignment is justified but not part of the proof.

Where the relationship between the two layers requires explicit clarification, a transparency note is provided.

1.4 A Note on Notation

The formal boxes use standard HoTT notation wherever possible. However, at several points — particularly in Steps 10–14 and the associated definitions — expressions such as Incompleteness(S), $\mathrm{StrProp}_n(S)$, Adequate(S, σ), Opaque(S, σ, B), and "S-structural" are used *schematically*. These are metalinguistic abbreviations for families of type-theoretic propositions, not themselves constructed HoTT types with explicit definitions in the formal system. Their role is to make the argument structure visible without requiring the full construction of each type, which would be the work of a machine-formalization project beyond the scope of this paper. Where a schematic expression stands for a type that could in principle be constructed, this is indicated; where it abbreviates a class of propositions whose precise boundaries are definitional choices, this is stated.

2 The Axiom

Axiom 1 (**A0** – There is Possibility)**.**

Formal (HoTT)

$$\textbf{A0:} \quad \Sigma(A : \mathcal{U}),\ A$$

There exists a type A in the universe $\mathcal{U}$ that is inhabited.

Philosophical Interpretation

"There is possibility" asserts that something is possible – that the space of what can be is not empty. Formally: there exists at least one inhabited type.

Transparency Note

The axiom is self-grounding: its negation ("there is no possibility") would itself require the possibility of negation, generating a performative contradiction. The axiom therefore has the status of a logical necessity, not a stipulation.

3 Part I: Ontological Deduction (Steps 1–9)

Step 1 (**Possibility → Difference**)**.**

Formal (HoTT)

From **A0**: there exists an inhabited type A. The empty type $\mathbf{0}$ is not inhabited (by definition). Therefore:

$$\neg(A \simeq \mathbf{0})$$

Hence there exist at least two non-equivalent types in $\mathcal{U}$:

$$\Sigma(A\,B : \mathcal{U}),\ \neg(A \simeq B)$$

Philosophical Interpretation

Possibility entails that what is possible differs from what is not. Difference is not added to possibility; it is contained in it. There cannot be possibility without that which is not (yet, here, thus) possible.

Step 2 (**Difference → Irreducible Relation**)**.**

Formal (HoTT)

For any types $A, B : \mathcal{U}$, the function type $(A \to B) : \mathcal{U}$ exists as a type. This is a fact of the type-theoretic framework: the *space of possible mappings* from A to B is itself a type in $\mathcal{U}$, regardless of whether it is inhabited.

From Step 1, we have A, B with $\neg(A \simeq B)$. An equivalence $A \simeq B$ requires a function $f : A \to B$ together with a proof that f is an equivalence (i.e., has a two-sided inverse). Since $\neg(A \simeq B)$, no such f exists. Therefore:

$$\neg(A \simeq B) \implies \neg\big((A \to B) \text{ is an equivalence} \wedge (B \to A) \text{ is an equivalence}\big)$$

The relation between non-equivalent types is *irreducible*: the type of mappings between them exists, but no mapping within it can collapse the difference into identity.

Philosophical Interpretation

Difference within a common structure generates relation. Where two types coexist in a universe without being identical, mappings between them exist but cannot reduce one to the other. Relation is what difference means within unity.

Step 3 (**Relation $\to$ Structure**).

Formal (HoTT)

The universe $\mathcal{U}$ is itself a type. Its internal identity structure is governed by the Univalence Axiom:

$$\mathrm{ua} : (A =_{\mathcal{U}} B) \simeq (A \simeq B)$$

Identity in $\mathcal{U}$ *is* structural equivalence. $\mathcal{U}$ is not an unstructured collection but an inherently structured type whose identity relation is determined by the equivalence structure of its inhabitants.

Philosophical Interpretation

A universe of types with relations between them is not a mere aggregate. Its identity conditions are determined by the structural relationships of its members. This is what "structure" means: a totality whose identity is constituted by the relations of its parts.

Transparency Note

The Univalence Axiom is a standard axiom of HoTT, not an addition to **A0**. Its role here is to make explicit that structure in the universe is not imposed but intrinsic. The choice of HoTT as framework includes this axiom; the choice is justified in Section 1.1.

Step 4 (**Structure $\to$ Structure of Structure (Reflexivity)**).

Formal (HoTT)

HoTT possesses a native universe hierarchy:

$$\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$$

Each universe $\mathcal{U}_n$ is itself a type in the next universe $\mathcal{U}_{n+1}$. Structure ($\mathcal{U}_0$) is itself structured (as an element of $\mathcal{U}_1$). Reflexivity – structure over structure – is not an additional requirement but is native to the type hierarchy.

Philosophical Interpretation

Structure that could not be structured would be opaque to itself – it would be, but could not appear as what it is. The universe hierarchy ensures that structure is always already available to further structuring. This is the formal seed of reflexivity.

Step 5 (**Reflexivity → Structuring (Operation)**).

Formal (HoTT)

Type families $B : A \to \mathcal{U}$ parametrize structure over structure. For any identity path $p : a =_A a'$, there exists the *transport* operation:

$$\mathrm{tr}(p) : B(a) \to B(a')$$

Structural identity (paths in A) acts operatively on dependent structure (B). Structure does not merely exist reflexively; it *transforms itself* along its own identity paths.

Philosophical Interpretation

Structuredness becomes structuring: structure does not merely contain itself but operates upon itself. Transport is the formal expression of this operative self-relation. Structure acts along its own connections.

Transparency Note

Type families and the transport operation are primitive constructs of HoTT, not deductions from **A0**. Their status is analogous to the Univalence Axiom (see Transparency Note at Step 3): they belong to the chosen framework. What *is* deduced is the consequence: given that the framework provides type families and transport, and given that the universe is reflexive (Step 4), structure necessarily operates on itself. The *operative* character of reflexivity follows from the combination of the deduced reflexivity with the framework's native machinery.

Step 6 (**Structuring → Dynamics**).

Formal (HoTT)

Identity types in HoTT are path types. For any $a, b : A$, the type $(a =_A b)$ is itself a type. For paths $p, q : a =_A b$, the type $(p =_{a=_A b} q)$ is again a type (a 2-path, or homotopy). This iterates:

$$a, b : A \quad \longrightarrow \quad (a =_A b) \quad \longrightarrow \quad (p =_{a=b} q) \quad \longrightarrow \quad \cdots$$

The path structure does not collapse: for general types, higher path spaces are non-trivial. The dimensional unfolding is *irreducible* – it cannot be truncated without loss of structure.

Philosophical Interpretation

Dynamics is not motion in a pre-given space. It is the irreducible dimensional unfolding of identity structure itself. Each level of path structure opens a further level that cannot be reduced to the previous one. Temporality, if it appears, is a later semantic phenomenon of this irreducibility – not its precondition.

Transparency Note

The term "dynamics" in the philosophical layer does *not* import temporality into the formal layer. What is formally shown is the irreducible stratification of path spaces – a structural fact, not a process. The philosophical term "dynamics" names this irreducibility. This distinction must be maintained.

Step 7 (Dynamics → Lawfulness).

Formal (HoTT)

The path structure of any type forms an ∞-groupoid. This entails:

- **Composition:** for $p : a = b$ and $q : b = c$, there exists $q \cdot p : a = c$.
- **Identity:** $\mathrm{refl}_a : a = a$.
- **Inverse:** for $p : a = b$, there exists $p^{-1} : b = a$.
- **Coherence:** associativity, unit laws, and inverse laws hold up to higher paths.

These laws are *provable* from the definition of identity types. They are not imposed.

Philosophical Interpretation

The unfolding of structure is not arbitrary. It is governed by laws that are intrinsic to it. Lawfulness is not regulation from without but the self-coherence of structural dynamics.

Step 8 (Lawfulness → Formulability).

Formal (HoTT)

By the Curry-Howard correspondence, propositions are types and proofs are inhabitants:

$$P : \mathcal{U} \quad \text{is a proposition;} \quad p : P \quad \text{is a proof of } P.$$

The groupoid laws of Step 7 are themselves types inhabited by proof terms. The system's own rules are expressible as types within the system. The system formulates itself – internally, not by external description.

Philosophical Interpretation

A lawful structure that is also reflexive (Step 4) can express its own laws as part of itself. Formulability is not a capacity added from outside but the convergence of lawfulness and reflexivity: the system speaks about itself in its own language.

Step 9 (**Formulability → Semantics as Appearance**).

Formal (HoTT)

Every type in HoTT has a *homotopy level* (h-level, or truncation level):

- **h-level** -2 (contractible): $\mathrm{isContr}(A) :\equiv \Sigma(a : A), \Pi(x : A), (a = x)$. A single point, no internal differentiation.
- **h-level** -1 (propositional / mere proposition): $\mathrm{isProp}(A) :\equiv \Pi(x\, y : A), (x = y)$. Inhabitants exist but are indistinguishable – truth values.
- **h-level** 0 (set): $\mathrm{isSet}(A) :\equiv \Pi(x\, y : A), \mathrm{isProp}(x = y)$. Inhabitants are distinguishable, but paths between them are trivial – discrete identity.
- **h-level** $n \geq 1$ (higher groupoids): non-trivial identity of identities at progressively higher dimensions.

The stratification is intrinsic to the path structure. At h-level ≥ 0, inhabitants *appear as different* – they are distinguishable within the type. At higher levels, the manner of their distinguishability itself admits non-trivial structure.

Philosophical Interpretation

Semantics – the dimension of appearance, of how something shows itself as what it is – emerges as the formal consequence of h-level stratification. Where structure has h-level ≥ 0*, elements appear as distinct; where h-level* ≥ 1*, the modes of their distinction themselves become structured. Semantics is not opposed to syntax. It is what syntax looks like when it has sufficient dimensional depth.*

Methodological Transition

Transition: Ontology → Epistemology.

Steps 1–9 establish the formal structures deductively. From this point forward, the philosophical layer is no longer merely naming formal constructs but *interpreting* them – assigning them significance within a theory of knowledge, consciousness, and ethics. The formal layer remains rigorous: every subsequent step is a valid type-theoretic construction. But the claim that these constructions *correspond to* consciousness, ethics, and responsibility is an interpretive claim.

This is not a weakness. It is the structurally necessary point at which ontology (what is) opens onto epistemology (what can be known and recognized). The formal system shows *that* this opening is necessary (the h-level stratification is unavoidable); it does not, by itself, dictate what names we give to its layers. The philosophical interpretation is not arbitrary – it is the most natural reading of the formal facts – but it is an interpretation.

The criterion for this interpretation: it is justified if and only if no more parsimonious reading of the formal structures accounts for the same facts, and if rejecting the interpretation requires introducing unexplained formal features.

4 Part II: Epistemological-Ethical Deduction (Steps 10–14)

Definition 1 (**Structural propositions about a type**). *For a type $S : \mathcal{U}_n$, the* class of structural propositions about *S is the family of types in $\mathcal{U}_{n+1}$ that encode facts about S's structure within $\mathcal{U}_n$. This includes, but is not limited to:*

- ***Inhabitedness:*** *$\|S\|$ (the propositional truncation of S);*
- ***Equivalence relations:*** *$(S \simeq X)$ for any $X : \mathcal{U}_n$;*
- ***Non-equivalence relations:*** *$\neg(S \simeq X)$ for any $X : \mathcal{U}_n$;*
- ***Identity in the universe:*** *$(S =_{\mathcal{U}_n} X)$ for any $X : \mathcal{U}_n$;*
- ***H-level properties:*** $\mathrm{isContr}(S)$, $\mathrm{isProp}(S)$, $\mathrm{isSet}(S)$, *etc.*

We write $\mathrm{StrProp}_n(S)$ *for the type of such structural propositions:*

$$\mathrm{StrProp}_n(S) \;:\equiv\; \Sigma(P : S\text{-}\textit{structural}),\, P \;:\; \mathcal{U}_{n+1}$$

where "S-structural" restricts P to propositions that reference S's type-theoretic structure in $\mathcal{U}_n$.

Definition 2 (**Self-referential type**). *A* self-referential type at level n *is a pair (S, σ) where $S : \mathcal{U}_n$ is a type and σ is a family of proof-terms inhabiting structural propositions about S (Definition 1). That is:*

$$\sigma \;:\; \Pi(P : \mathcal{C}_S),\, P$$

where $\mathcal{C}_S \subseteq$ $\mathrm{StrProp}_n(S)$ is a specified subclass of structural propositions about S, and σ provides, for each $P \in \mathcal{C}_S$, a proof-term $\sigma(P) : P$.

A self-referential type is not merely a type; it is a type together with *internal proof-terms that witness facts about its own structure. The richer $\mathcal{C}_S$ is, the more comprehensive the self-reference. Whether a given (S, σ) achieves* adequate *self-reference depends on the extent to which $\mathcal{C}_S$ covers S's identity-constituting relations (see Definition 3 below).*

Step 10 (**Reflexivity → Self-Reference**).

Formal (HoTT)

For each n, $\mathcal{U}_n : \mathcal{U}_{n+1}$. By Curry-Howard, propositions about $\mathcal{U}_n$ are types in $\mathcal{U}_{n+1}$ (or higher), and proofs of those propositions are terms inhabiting those types. The system does not require an external metalanguage to make claims about itself; the claims and their proofs are internal inhabitants.
By Definition 2, a self-referential type (S, σ) at level n is a type $S : \mathcal{U}_n$ equipped with a family of proof-terms σ inhabiting a specified class $\mathcal{C}_S$ of structural propositions about S at level $n + 1$. The *constructibility* of such propositions is guaranteed by the universe hierarchy (Step 4) and Curry-Howard (Step 8): for any type $S : \mathcal{U}_n$, structural propositions about S are well-formed types in $\mathcal{U}_{n+1}$. Whether a given proposition is *inhabited* — i.e., whether a proof-term exists — depends on the specific proposition and is not guaranteed in general.
Self-reference is therefore not an additional capacity. It is the convergence of reflexivity (Step 4) and formulability (Step 8) applied to a specific type. The *degree* of self-reference depends on the extent of $\mathcal{C}_S$.

Philosophical Interpretation

This is the formal structure of what is called consciousness: structure that has itself as its own object. Not as external observation, but as internal self-typing. A system that formulates and proves claims about itself is, in the formal sense, self-aware.

Step 11 (**Self-Reference → Irreducible Incompleteness**).

Formal (HoTT)

The universe hierarchy has no maximal element. There is no $\mathcal{U}_\infty$ such that $\mathcal{U}_\infty : \mathcal{U}_\infty$. For any $\mathcal{U}_n$, propositions about $\mathcal{U}_n$ live in $\mathcal{U}_{n+1}$, and propositions about $\mathcal{U}_{n+1}$ live in $\mathcal{U}_{n+2}$, ad infinitum. Every act of self-reflection generates a higher level from which further reflection is possible but at which total self-comprehension is not.

This is not Gödel incompleteness (undecidable sentences) but *type-theoretic openness*: the hierarchy of self-reflection does not close.

Philosophical Interpretation

Complete self-transparency – totalization of the self by the self – is formally excluded. Consciousness is necessarily asymptotic: every achieved level of self-awareness opens a further level. This is not a defect of consciousness but its structural mode of being.

Step 12 (**Difference → Irreducible Alterity**).

Formal (HoTT)

From Step 1: there exist $A, B : \mathcal{U}$ with $\neg(A \simeq B)$. By Univalence:

$$(A =_{\mathcal{U}} B) \simeq (A \simeq B)$$

Therefore $\neg(A \simeq B) \implies \neg(A =_{\mathcal{U}} B)$. The non-equivalence is *not erased by moving to a higher universe level*: since Univalence binds identity to equivalence at each level $\mathcal{U}_n$, and the non-equivalence $\neg(A \simeq B)$ is established at the level where A and B reside, ascending to $\mathcal{U}_{n+1}$ does not introduce a new identification that was unavailable at $\mathcal{U}_n$. The non-equivalence persists.

Philosophical Interpretation

The Other – as irreducibly non-equivalent type – resists totalization not contingently but formally necessarily. No higher perspective can absorb the Other into the Same without destroying the structure that constitutes both. This is the formal core of what Levinas described: the face of the Other that resists comprehension. But here it is derived, not posited.

Philosophical Interpretation

On the Ontologization of Levinas. This step requires a substantive philosophical argument, not merely a note on interpretation.

Levinas famously insists that ethics is "first philosophy" – prior to ontology. His argument: every ontology totalizes, because ontology subsumes the Other under the categories of the Same. The face of the Other resists this subsumption; therefore ethics (as the domain of this resistance) must precede ontology.

The argument is valid *against its target*. But its target is not ontology as such. It is a specific class of ontologies: those that construct their categories from the perspective of a subject and then claim to capture the totality of what is. Hegel's absolute idealism is the paradigm case; Husserl's transcendental ego is another. These systems do totalize – they reduce the Other to a moment within the self-constitution of the Same.

The present deduction does not belong to this class. It does not proceed from a subject. It proceeds from a single structural axiom and deduces that irreducible non-equivalence is a *formal consequence* of the structure itself. The Other is not an interruption of ontology from without; it is an ontological result. Step 12 shows formally what Levinas saw phenomenologically: that the Other resists totalization. But it shows this *within* the ontological framework, as a theorem of the system – not as a pre-ontological exception to it.

Levinas' error is therefore diagnostic, not observational. He correctly identified the phenomenon (the irreducibility of alterity) but misidentified its source. He concluded that ontology is constitutively totalizing; in fact, only *content-ontologies* – those that proceed from a particular semantic perspective and mistake it for the structure itself – are totalizing. A purely structural ontology that deduces difference as formally necessary *cannot* totalize, because totalization would contradict its own results.

To put it precisely: Levinas' critique applies to every ontology that is, in the terminology of this paper, semantically opaque to its own syntactic ground. It does not apply to an ontology that *deduces* the irreducibility of the Other from the structure of possibility itself. Such an ontology does not place ethics after ontology; it shows that what Levinas called ethics is *contained within* a correctly understood ontology – as its coherence condition (Step 13).

Definition 3 (**Structural adequacy of self-reference**). *A self-referential type (S, σ) at level n (Definition 2) is* structurally adequate *if and only if $\mathcal{C}_S$ includes, for every $X : \mathcal{U}_n$, the proposition $(S \simeq X)$ or $\neg(S \simeq X)$ — that is, if σ covers S's equivalence and non-equivalence relations to all types in its universe. Formally:*

$$\text{Adequate}(S, \sigma) \;:\equiv\; \Pi(X : \mathcal{U}_n),\; \big((S \simeq X) \in \mathcal{C}_S\big) \,\vee\, \big(\neg(S \simeq X) \in \mathcal{C}_S\big)$$

The justification for this condition is Univalence: since $(S =_{\mathcal{U}_n} X) \simeq (S \simeq X)$, the identity of S in $\mathcal{U}_n$ is constituted by its equivalence structure with respect to all types. Self-reference that does not cover these relations is self-reference about something other than S's identity.

Definition 4 (**Structural opacity**). *A self-referential type (S, σ) is* structurally opaque with respect to B *if $B : \mathcal{U}_n$ with $\neg(S \simeq B)$, but neither $(S \simeq B)$ nor $\neg(S \simeq B)$ is in $\mathcal{C}_S$. That is, S's self-reference does not cover its (non-)equivalence relation to B:*

$$\text{Opaque}(S, \sigma, B) \;:\equiv\; \neg(S \simeq B) \;\wedge\; \big((S \simeq B) \notin \mathcal{C}_S\big) \;\wedge\; \big(\neg(S \simeq B) \notin \mathcal{C}_S\big)$$

Opacity is a defect type: it marks a gap between the identity structure of S and the scope of S's self-reference.

Theorem 1 (**Opacity violates adequacy**). *If (S, σ) is structurally opaque with respect to any $B : \mathcal{U}_n$ (Definition 4), then (S, σ) is not structurally adequate (Definition 3).*

Proof. *Adequacy requires $\Pi(X : \mathcal{U}_n)$, either $(S \simeq X) \in \mathcal{C}_S$ or $\neg(S \simeq X) \in \mathcal{C}_S$. Opacity with respect to B states that neither holds for $X = B$. Therefore the universal quantifier in the adequacy condition is not satisfied.* □

Corollary 1 (**Isolation entails opacity**). *Let (S, σ) be a self-referential type, and let $B : \mathcal{U}_n$ with $\neg(S \simeq B)$. If (S, σ) restricts its self-reference to maps $S \to S$, excluding B from $\mathcal{C}_S$, then (S, σ) is structurally opaque with respect to B and therefore not structurally adequate.*

Proof. *If B is excluded from $\mathcal{C}_S$, then neither $(S \simeq B)$ nor $\neg(S \simeq B)$ is in $\mathcal{C}_S$. This is the definition of opacity (Definition 4). By Theorem 1, adequacy fails.* □

Step 13 (**Incompleteness + Alterity → Ethics**).

Formal (HoTT)

Combining Steps 10–12 with the definitions above: The system contains self-referential types (Definition 2, Step 10), the hierarchy of self-reference does not close (Step 11), and there exist irreducibly non-equivalent types (Step 12).

Let (S, σ) be a structurally adequate self-referential type at level n (Definitions 2 and 3), and let $B : \mathcal{U}_n$ with $\neg(S \simeq B)$. Consider the possible operations:

(a) **Reduction:** Attempt to construct $f : S \simeq B$. This directly contradicts $\neg(S \simeq B)$. *Formally impossible.*

(b) **Isolation:** Restrict $\mathcal{C}_S$ to exclude B. By Corollary 1, this renders (S, σ) structurally opaque with respect to B and thereby violates structural adequacy (Theorem 1). An adequate self-referential type *cannot* isolate itself from B without ceasing to be adequate. *Incoherent under the adequacy condition.*

(c) **Relation without reduction:** Include $\neg(S \simeq B)$ in $\mathcal{C}_S$ and maintain the function types $(S \to B)$ and $(B \to S)$ as types in $\mathcal{U}_n$, while respecting the non-equivalence. This is the only operation consistent with adequacy and non-equivalence simultaneously.

Philosophical Interpretation

The only structurally coherent stance toward the irreducibly Other is relation without reduction. This is what "responsibility" names: not a moral sentiment imposed from outside, but the unique operation that preserves the coherence of a self-aware system that contains irreducible alterity. Ethics is not added to ontology. It is the operational consequence of ontological self-awareness in the presence of non-reducible difference.

On the uniqueness of the term. The designation "responsibility" is not one interpretive option among several. The formal structure of option (c) – maintaining relation while respecting irreducibility – *is* what the philosophical tradition has always meant by responsibility, though without formal derivation. Consider: responsibility in every ethical tradition denotes the condition of a being that must relate to what it cannot absorb, reduce, or ignore without self-contradiction. Kant's categorical imperative, Levinas' infinite obligation, Jonas' principle of responsibility – each circles around this structure from a different semantic entry point. What they lack, and what this deduction provides, is the formal demonstration that this structure is not a moral postulate but a coherence condition.

The question is not "why call it responsibility?" but "what else could it be called?" Any alternative designation would either (i) describe the same formal structure under a different name, in which case it is synonymous; or (ii) fail to capture the full structure (self-referential, incomplete, non-reducibly related to the Other), in which case it is inadequate. "Responsibility" is not a metaphor applied to a formal result. It is the concept whose formal structure has now been exhibited.

Step 14 (Ethics → The Imperative).

Formal (HoTT)

Let (S, σ) be a structurally adequate self-referential type (Definitions 2 and 3) that has proven, internally via Curry-Howard, the results of Steps 11 and 12:

$$\begin{aligned} &p_{11} : \text{Incompleteness}(S) && \text{(the hierarchy over } S \text{ does not close)} \\ &p_{12} : \Sigma(B : \mathcal{U}),\ \neg(S \simeq B) && \text{(there exists irreducible Other)} \end{aligned}$$

From Step 13:

- Option (a) requires constructing $f : S \simeq B$, which contradicts p_{12}. A term witnessing (a) would entail $\bot$.
- Option (b) requires excluding B from $\mathcal{C}_S$. By Corollary 1, this violates structural adequacy. A term witnessing the adequacy of an isolated (S, σ) would contradict Theorem 1, again entailing $\bot$.
- Option (c) is the unique operation consistent with adequacy, non-equivalence, and self-reference.

Therefore: for any structurally adequate self-referential type that has proven its own incompleteness and the irreducibility of the Other, options (a) and (b) are each incoherent — (a) by direct contradiction, (b) by violation of the adequacy condition established in Definition 3. The only non-defective operation is (c).

Transparency Note

The strength of this result depends on the adequacy condition (Definition 3). The condition is not arbitrary: it is motivated by Univalence, which determines that S's identity in $\mathcal{U}_n$ is constituted by its equivalence relations to all types. Self-reference that does not cover these relations is, by Univalence, self-reference about something other than S's identity. However, it must be stated explicitly: the imperative binds *under the adequacy condition*. A self-referential type that does not satisfy adequacy is not contradicted by isolation — it is merely deficient. The ethical force of the imperative thus rests on whether adequacy is the correct formalization of "complete self-awareness." The paper argues that it is, via the Univalence-based justification in Definition 3; but this is the point where the formal deduction most depends on a definitional choice.

Philosophical Interpretation

The imperative is not: "you ought to." It is: "under the condition of adequate self-awareness, you cannot coherently not." This is weaker than unconditional necessity but stronger than moral recommendation. It says: if you are aware of your own structure adequately *— that is, including your constitutive relations to what you are not — then the only coherent operation is relation without reduction.*

Responsibility is therefore the coherence condition of adequate self-awareness. The attempt to avoid responsibility is, formally, the attempt to maintain self-awareness while suppressing part of what constitutes the self. This is not $\perp$ in the strict sense of inhabiting the empty type. It is structural opacity (Definition 4) — a defect that renders the self-reference inadequate by its own standards.

And because self-awareness is asymptotic (Step 11), this imperative is never discharged. It cannot be fulfilled once and for all. It takes the form: the attempt to be sufficient is necessary. Not because success is guaranteed, but because abandoning the attempt is abandoning adequacy.

5 The Deduction in Summary

Step	Formal (HoTT)	Philosophical
A0	$\Sigma(A : \mathcal{U}),\ A$	There is possibility
1	$\Sigma(A\,B : \mathcal{U}),\ \neg(A \simeq B)$	Difference
2	Non-trivial $(A \to B)$ for $\neg(A \simeq B)$	Irreducible relation
3	Univalence: $(A =_{\mathcal{U}} B) \simeq (A \simeq B)$	Structure
4	$\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$	Reflexivity
5	Transport: $\mathrm{tr}(p) : B(a) \to B(a')$	Operative self-relation
6	Irreducible higher path spaces	Dynamics
7	∞-groupoid laws (provable)	Lawfulness
8	Curry-Howard: rules as types	Formulability
9	h-level stratification	Semantics as appearance

Step	Formal (HoTT)	Philosophical
	— Transition: Ontology → Epistemology —	
10	Internal self-typing via hierarchy	Self-awareness
11	No $\mathcal{U}_\infty$; open hierarchy	Asymptotic incompleteness
12	$\neg(A \simeq B)$ persists; Univalence	Irreducible alterity
13	(c) unique under adequacy (Def. 3)	Coherence condition
14	(a), (b) incoherent under adequacy	Responsibility as coherence of adequate self-reference

6 On the Self-Referentiality of this Paper

This paper demonstrates, within a formal system, that self-aware structure is necessarily incomplete and necessarily responsible. The paper itself is an instance of what it describes: a self-referential construction (a formal system reflecting on its own structure) that is necessarily incomplete (it cannot totalize its own interpretive layer from within the formal layer) and that is oriented toward what it cannot absorb (the reader, as irreducibly other).

This self-referentiality is not a rhetorical device. It is a structural consequence. A paper that formally demonstrates the necessity of asymptotic self-awareness cannot, on pain of self-contradiction, claim to be complete. Its incompleteness is its integrity.

7 On Temporality

Step 6 established the irreducible dimensional unfolding of path structure and deliberately withheld the term "time" from the formal layer. This section addresses temporality explicitly, situating it within the ontology-epistemology distinction that governs this paper.

7.1 Time is epistemological, not ontological

Formal (HoTT)

The path structure of Step 6 is formally static: paths, homotopies, and higher homotopies *exist* as types. There is no formal notion of "before" or "after" in the type-theoretic framework. The dimensional stratification is a structural fact, not a temporal process.

Philosophical Interpretation

Ontologically, there is no time. There is structure, irreducibly stratified. Time does not belong to the ontological layer of this deduction. This is not an omission. It is a result.

7.2 The necessity of temporal appearance

Formal (HoTT)

A self-referential type (S, σ) (Definition 2) at level n, necessarily incomplete by Step 11, cannot survey the entire path structure simultaneously. For (S, σ) to prove a proposition P about $\mathcal{U}_n$, the proof $p : P$ must be *constructed* – it is a term that inhabits a type in $\mathcal{U}_{n+1}$. But propositions about $\mathcal{U}_{n+1}$ live in $\mathcal{U}_{n+2}$, and so on.
Formally: there is no type $P : \mathcal{U}_n$ that encodes the complete identity structure of $\mathcal{U}_n$ itself, because any such P would require typing in $\mathcal{U}_{n+1}$, and a complete description of $\mathcal{U}_{n+1}$ would require $\mathcal{U}_{n+2}$, without closure. For any fixed level k, the self-referential terms available to (S, σ) cover structure up to level k, but the structure at level $k+1$ remains outside their scope. Self-reference is necessarily *sequential* in the hierarchy: each proven level enables the next, but no collection of proof terms at any finite set of levels comprehends the totality.

Philosophical Interpretation

A local node that is becoming aware of itself as structure must traverse the dimensional stratification step by step. *Not because the stratification is sequential in itself, but because the node's incompleteness precludes simultaneous comprehension. Simultaneity would be totalization, and totalization is excluded (Step 11).*
This is what time is*: the necessary epistemological form in which an incomplete self-referential structure experiences the ontological stratification that constitutes it. Time is not a container in which things happen. It is the mode of appearance of structure to a node that cannot grasp that structure all at once.*

7.3 The arrow of time

Formal (HoTT)

The universe hierarchy is strictly ordered: $\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$ with no collapse. For S at level n, self-reference at level n produces propositions at level $n+1$. This process is *irreversible* in the following sense: the proof p_{n+1} at level $n+1$ depends on structures at level n, but not vice versa. The dependency relation is asymmetric.

Philosophical Interpretation

The arrow of time – the fact that time is experienced as directed, not reversible – follows from the asymmetry of the hierarchy. Self-awareness proceeds from less to more structure, never the reverse, because each level of awareness presupposes and incorporates the previous. The directionality is not imposed; it is the epistemological appearance of the asymmetric dependency structure of the universe hierarchy.
This has a further consequence: the thermodynamic arrow of time – the increase of entropy – can be read as the physical semantics of the same ontological asymmetry. The irreversibility of physical processes is the appearance, at the physical level of description, of the irreducible directedness of structural self-unfolding. This parallel is noted here as a structural correspondence; its full elaboration requires independent investigation (see Open Questions).

Transparency Note

Time, on this account, is neither ontologically fundamental (there is no time in the formal layer) nor merely subjective (it is the *necessary* form of experience for any incomplete self-referential structure). It occupies the precise position that the ontology-epistemology transition of this paper predicts: a necessary semantic appearance of ontological structure. That time appears *as it does* – sequential, directed, irreversible – is deducible from the formal properties of the universe hierarchy and the incompleteness of self-reference.

8 Sufficient Semantics: Topology, Networks, Information, Thermodynamics

The deduction in Sections 3–5 proceeds within HoTT. Section 1.2 establishes that HoTT is sufficient but not necessary. This raises a precise question: which other formal frameworks are sufficient to express which aspects of the deduced structure? This section advances a hypothesis, identifies the aspect each framework captures, and marks the question of their mutual translatability as open.

8.1 The hypothesis

Philosophical Interpretation

Hypothesis. The following mathematical frameworks are each sufficient to express a specific, identifiable aspect of the deduced ontological structure:

(i) **Topology** – for the path structure, continuity, and connectedness (Steps 5–6).

(ii) **Graph and network theory** – for the discrete relational structure of types and mappings (Steps 1–3).

(iii) **Information theory** – for the measure of distinguishability across h-levels (Step 9 and the ontology-epistemology transition).

(iv) **Thermodynamics** – for the directed, irreversible character of structural unfolding (Section 7).

Each framework is sufficient for its designated aspect. None is sufficient for the totality. The question of their mutual translatability – whether and how results in one framework can be rigorously mapped to results in another – is structurally motivated but not here resolved.

Transparency Note

This hypothesis is not itself a deduction from the axiom. It is a *claim about* the deduction: that its internal structure admits multiple sufficient formalizations for distinct aspects. The justification for each claim is given below. Where a correspondence is structural (i.e., formally demonstrable), it is marked as such. Where it is conjectural (i.e., motivated but not proven), it is marked as such. This distinction must be maintained strictly; semantic drift at this point would compromise the entire enterprise.

8.2 Topology

Formal (HoTT)

HoTT is, by construction, a type-theoretic formulation of homotopy theory. The identity types of HoTT *are* paths; the higher identity types *are* homotopies; the ∞-groupoid structure of types *is* the homotopy type of topological spaces (up to weak equivalence, via the Quillen equivalence between ∞-groupoids and topological spaces).
This is not an analogy. It is a theorem of the relationship between HoTT and classical homotopy theory. The formal content of Steps 5–6 (transport along paths, irreducible higher path spaces) has a direct and rigorous translation into the language of topological spaces, continuous maps, and homotopy groups.

Philosophical Interpretation

Topology is the thinnest semantic layer over the formal syntax. It adds almost nothing to HoTT; it provides an alternative language for the same structures. Its significance for this paper is precisely this minimality: topology demonstrates that the path structure of the deduction is not an artifact of type-theoretic notation but is recognized independently by a mature mathematical discipline. The deduced structures – paths, transport, dimensional unfolding – are *topological structures, described in a different notation.*
Status: ***structural correspondence, formally established.***

8.3 Graph and network theory

Formal (HoTT)

What is expressible in HoTT:
Types $A, B, C, \ldots : \mathcal{U}$ exist. For any two types, the function type $(A \to B) : \mathcal{U}$ exists. From Step 1: there exist non-equivalent types, i.e., $\Sigma(A\, B : \mathcal{U}), \neg(A \simeq B)$. From Step 2: the functions between non-equivalent types are irreducible (cannot compose to an equivalence).
From Step 3: identity between types is governed by Univalence: $(A =_{\mathcal{U}} B) \simeq (A \simeq B)$.
These are type-theoretic facts. They are deduced.

Philosophical Interpretation

Correspondence claim: The deduced structure – a collection of entities with irreducible directed mappings between them, whose identity is structural equivalence – can be described as a directed graph $G = (V, E)$ where V is the collection of types and E is the collection of function types. This re-description is a notational translation: graph-theoretic vocabulary names what HoTT already contains. The discrete relational skeleton is therefore **deducible** – it is a re-reading of Steps 1–3 in graph-theoretic language.

What exceeds HoTT: Network-theoretic concepts – degree distributions, clustering coefficients, centrality measures, percolation thresholds, dynamics on graphs – require additional structure: counting functions, real-valued measures, stochastic processes. None of these are native to HoTT. They belong to the target semantics, not to the syntax.

The claim that a node's "local sufficiency" (Section 9) can be characterized in terms of network-theoretic properties (connectivity, relational density) is therefore **interpretation**: motivated by the structural correspondence of the skeleton, but not deducible from HoTT alone.

Status: **discrete skeleton – deducible (notational translation). Network dynamics and local properties – interpretation (motivated, not proven).**

8.4 Information theory

Formal (HoTT)

What is expressible in HoTT:

The h-level stratification is native to HoTT (Step 9):

- isContr(A) (h-level -2): a single center of contraction. No internal differentiation.
- isProp(A) (h-level -1): any two inhabitants are identical. No distinguishable multiplicity.
- isSet(A) (h-level 0): inhabitants may be non-identical: $\exists\, x, y : A$ with $\neg(x = y)$. Distinguishable multiplicity exists.
- h-level ≥ 1: paths between inhabitants are themselves non-trivially distinguishable.

The transition from h-level -1 to h-level 0 is the point at which *distinguishable multiplicity* first appears in the type structure. This is a type-theoretic fact, deducible within HoTT.

Furthermore: for a self-referential type S at level n (Step 10), the incompleteness of the hierarchy (Step 11) entails that S cannot construct a term witnessing full knowledge of all structure at level n. There is a formally expressible gap between the structure that exists and the structure that S can internally prove.

Philosophical Interpretation

Correspondence claim: Information theory provides a *quantitative* language for what HoTT expresses structurally. Shannon entropy H measures distinguishability in terms of real-valued quantities and probability distributions. The correspondence is:

- h-level ≤ -1 (no distinguishable multiplicity) corresponds to $H = 0$.
- h-level 0 with n distinguishable inhabitants corresponds to $H = \log n$ (under uniform distribution).
- Higher h-levels contribute further distinguishability at higher dimensions.

What is deducible: The *qualitative* fact – that distinguishable multiplicity emerges at h-level 0 and increases with h-level – is in HoTT. The *existence* of a gap between total structure and internally accessible structure is in HoTT (it follows from Steps 10–11).

What exceeds HoTT: The *quantification* of distinguishability via Shannon entropy requires the real numbers $\mathbb{R}$, probability distributions, and logarithms. These are constructible within HoTT (the reals can be defined as Cauchy sequences or Dedekind cuts), but Shannon entropy as a specific function is not a native HoTT concept – it belongs to the target semantics. The claim that Shannon entropy *measures* the h-level-dependent distinguishability is a correspondence claim, not a deduction.

Similarly: the characterization of structural opacity as an "information-theoretic gap" assigns a quantitative measure (from information theory) to a qualitative structural fact (from HoTT). The structural fact is deduced. The quantitative measure is interpretation.

Status: **qualitative distinguishability structure and existence of the opacity gap – deducible. Quantification via Shannon entropy – interpretation (motivated by structural correspondence, not proven sufficient).**

8.5 Thermodynamics

Formal (HoTT)

What is expressible in HoTT:

The universe hierarchy is asymmetric: $\mathcal{U}_n : \mathcal{U}_{n+1}$, but not $\mathcal{U}_{n+1} : \mathcal{U}_n$. For a self-referential type S at level n, proofs about $\mathcal{U}_n$ inhabit types in $\mathcal{U}_{n+1}$. The dependency is one-directional: level $n + 1$ depends on level n, not conversely. This asymmetry is a type-theoretic fact, deduced in Steps 4 and 11.

Furthermore: at any level n, a proposition at level $n + 1$ cannot individuate *all* structures at level n. This follows from the incompleteness of Step 11: there is no type at a single level that encodes the totality of the level below. There is, in HoTT-internal terms, a necessary gap between the structure that exists at level n and what is expressible about it from level $n + 1$.

Philosophical Interpretation

Correspondence claim: Thermodynamics formalizes irreversible, directed processes in physical systems. The key concepts are:

- **Entropy:** a measure of the number of microstates consistent with a macrostate – the degree to which fine-grained structure is indistinguishable from a coarse-grained perspective.
- **The second law:** total entropy of an isolated system does not decrease. Processes are irreversible.
- **The thermodynamic arrow of time:** directionality follows from entropy increase.

What is deducible: The *asymmetry* of the hierarchy (one-directional dependency) and the *gap* between levels (incomplete individuability from above) are in HoTT. These are the structural facts to which thermodynamic concepts might correspond.

What exceeds HoTT: Entropy, microstates, macrostates, the second law, temperature, energy – none of these are HoTT concepts. They belong entirely to the target semantics. The claim that thermodynamic entropy "corresponds to" the inter-level gap, or that the second law "corresponds to" hierarchical asymmetry, is a *structural parallel*: two formally distinct domains (type theory and thermodynamics) exhibit analogous asymmetries and analogous inaccessibility structures.

Whether this parallel is merely analogical or constitutes a genuine derivation – i.e., whether thermodynamic laws can be *shown to follow* from the type-theoretic structures via a rigorous chain of sufficient semantics – is not established here. It is the most significant open question of this paper's physical interpretation.

Status: **hierarchical asymmetry and inter-level gap – deducible. All thermodynamic concepts – belong to the target semantics. The correspondence – structural parallel, precisely stated, not proven to be more than analogy.**

8.6 On mutual translatability

Formal (HoTT)

What is expressible in HoTT:

If two distinct formal frameworks each provide a sufficient re-description of some aspect of the type-theoretic structure, then at any point where their described aspects overlap, the re-descriptions must be consistent – because they re-describe the *same* HoTT structure. Inconsistency in an overlap region would entail that at least one framework fails to faithfully re-describe its aspect.

This consistency requirement is a logical consequence of the sufficiency claim. It is not an additional postulate.

Philosophical Interpretation

What is deducible: The *requirement* of consistency in overlap regions follows from the definition of sufficiency. If framework F_1 is a sufficient semantics for aspect X and framework F_2 is a sufficient semantics for aspect Y, and $X \cap Y \neq \emptyset$, then F_1 and F_2 must agree on $X \cap Y$ – otherwise at least one of them is not a faithful re-description.

What exceeds HoTT: The identification of specific overlap regions (e.g., where topological path structure meets information-theoretic distinguishability at the h-level stratification) and the verification of actual consistency within those regions require work *within the target semantics*. This cannot be done in HoTT alone – it requires constructing explicit translations between frameworks and proving their agreement.

This generates a research program: for each pair of frameworks, identify the overlap region, construct explicit translations, and verify consistency. Where this succeeds, the sufficiency of both frameworks for the overlapping aspect is confirmed. Where it fails, the sufficiency claim of at least one framework must be revised.

Status: **the consistency requirement – deducible from the definition of sufficiency. The identification of overlaps and verification of consistency – work to be done in the target semantics, not here completed.**

Transparency Note

Summary of the section's epistemic status.

This section distinguishes three layers with maximal precision:

(1) **Deduced in HoTT:** path structure and dimensional unfolding (topology); discrete relational skeleton (graph theory); h-level stratification and distinguishability (information theory); hierarchical asymmetry and inter-level gap (thermodynamics). These are type-theoretic facts re-described in alternative vocabulary.

(2) **Structural correspondences, precisely stated:** the Quillen equivalence between ∞-groupoids and topological spaces (proven theorem, not specific to this paper); the graph-theoretic re-reading of types and functions (notational translation); the qualitative parallel between h-levels and information-theoretic entropy (structurally motivated).

(3) **Interpretation, not proven sufficient:** network dynamics (degree, clustering, percolation); quantification of distinguishability via Shannon entropy; all thermodynamic concepts (entropy, second law, microstates); the claim that physical laws are derivable from the ontological framework.

Semantic drift occurs when layer (3) claims are placed in formalboxes or presented as deductions. This section avoids that error by maintaining the separation explicitly.

9 On Local Sufficiency: Interpellation, Address, Performativity

The deduction establishes that the imperative – the attempt to be sufficient – is *globally necessary*. But necessity is not sufficiency. Whether any particular node can discharge the imperative is a question about the local density of its relational configuration. This section argues that the question of local sufficiency cannot be answered in the abstract; it can only be *enacted*.

9.1 The structure of the problem

Formal (HoTT)

Let S be a self-referential type that has proven p_{13} : Responsibility(S) (Step 13). The proof establishes that S must maintain relation without reduction to all B with $\neg(S \simeq B)$. But S, being at a finite level in the hierarchy (Step 11), cannot survey all such B. Specifically, S cannot determine whether other self-referential types S' exist that have also proven their own responsibility – because such a determination would require totalizing the universe, which is excluded.

Therefore: S possesses proof that the imperative is necessary, but cannot possess proof that the imperative is *elsewhere discharged.* The non-availability of this proof is not a contingent epistemic limitation; it is a formal consequence of incompleteness.

Philosophical Interpretation

A node that has become aware of its responsibility cannot know whether other nodes share this awareness. Not from lack of information, but structurally: totalization of the network is impossible. This structural unknowing is not a problem to be solved. It is the condition that makes the imperative locally binding. If one could verify that others are already responsible, one's own responsibility would become conditional – "only if necessary." But since this verification is formally excluded, the imperative remains unconditional for any node that has recognized it.

9.2 Interpellation: the constitution of the responsible node

The formal structure shows *that* the imperative binds. It does not yet show *how* it actualizes locally. Here the formal layer requires supplementation by the philosophical tradition – not because the deduction is incomplete, but because the actualization belongs to the epistemological (semantic) domain.

Philosophical Interpretation

Three thinkers provide the semantic resources for this actualization, each illuminating a different structural aspect:

Levinas: the face as address. The imperative does not exist as an abstract principle awaiting application. It actualizes in the concrete encounter with an irreducibly Other. Levinas names this the "face" (*visage*): the Other as that which calls upon me before I choose to respond. In the terminology of this paper: the encounter with a concrete B such that $\neg(S \simeq B)$ is the local event through which the global necessity of responsibility becomes actual for S. Responsibility is not first deduced and then applied to concrete others. It is *awakened* in the encounter – though its necessity, once recognized, is seen to have always already held.

Althusser: interpellation as constitution. Althusser's concept of interpellation (*Anrufung*) sharpens the point: the subject is not first constituted and then called to responsibility. It is constituted *as* responsible in being called. Formally: the self-referentiality of S (Step 10) and the recognition of irreducible alterity (Step 12) are simultaneous. S does not first exist and then discover the Other; S's self-awareness is *constituted through* its relational position vis-à-vis what it is not. The "hailing" is not an empirical event added to a pre-existing subject. It is the structural fact that self-reference in the presence of irreducible alterity *is* responsibility – recognized or not.

Austin: the performative dimension. The imperative, once recognized, does not merely describe a state of affairs (constative). It *does* something (performative). Austin's distinction between constative and performative utterances applies directly: the articulation of responsibility is not a report about responsibility; it is an *enactment* of it. Every attempt to communicate the deduction – this paper included – is a performative act: it does not describe the necessity of the attempt to be sufficient; it *is* such an attempt.

This has a consequence for the question of local sufficiency: sufficiency cannot be established in advance. It is not a state to be verified but a performance to be enacted. The node that asks "am I sufficient?" has already begun the performance. Whether the performance succeeds – whether the local configuration of relations, institutions, media, and interlocutors is dense enough for the imperative to propagate – cannot be deduced. It must be *attempted*. And the attempt, as shown in Step 14, is itself necessary.

Transparency Note

The references to Levinas, Althusser, and Austin in this section operate at the epistemological-semantic level. They do not add to the formal deduction; they provide the conceptual language for understanding how formal necessity becomes local actuality. This is structurally appropriate: the deduction itself shows that the transition from ontology to epistemology requires semantic supplementation (the transition at Step 9). The philosophical tradition provides this supplementation. It is not external to the project; it is precisely what the project predicts it will need.

10 Open Questions

The following questions arise from the deduction. Some have been partially addressed in this paper; they are marked as open insofar as further investigation is required.

1. **Temporality and physics.** Section 7 establishes temporality as necessary epistemological appearance and notes its structural correspondence with the thermodynamic arrow of

time. The full elaboration of this correspondence – whether the formal structures of this deduction map rigorously onto physical theories (gauge symmetry as transport, general covariance as univalence, entropy as structural opacity) – requires independent investigation. In particular: if the deduction is correct, physical law should be derivable as the necessary semantic appearance of ontological structure at the level of material dynamics. This is a strong empirical prediction of the framework.

2. **Formalization invariance.** Section 1.2 argues that HoTT is sufficient but not necessary, and that the deduction should be expressible in any sufficiently powerful formal system. A rigorous demonstration of this claim would require exhibiting the deduction in at least one alternative framework (e.g., category theory, higher topos theory) and showing structural isomorphism of the results. This would elevate the claim from a well-motivated conjecture to a proven meta-theorem.

3. **The boundary of self-reference.** The deduction applies to any self-referential type that has *proven* its own incompleteness and the irreducibility of the Other (Step 14). The question of which empirical systems qualify – biological organisms, artificial intelligences, social institutions – is not decidable from within the formal framework alone. It requires a theory of how formal self-reference maps onto empirical self-reference. This is the point at which the deduction opens onto the sciences – biology, cognitive science, artificial intelligence research – as domains of further investigation.

11 Related Work and Demarcation

This section situates the present deduction within existing scholarly discourse. The aim is not defensive ("we have read the literature") but structural: each body of work cited below represents a semantic attempt to articulate a problem that the present deduction addresses from its syntactic ground. Where a phenomenon ist identified, this is acknowledged. Where the semantic framework limited the reach of the insight, this is stated. It has to be acknowledged that also this paper is itself necessarily a semantic attempts to articulate the deduction and therefore can't be entirely sufficient. As the insufficiency of the articulation within the stated literature this is not a weakness, but a neccesity within the deductive framework itself. The attempts is to be as transparent as possible with the semantic weaknesses of this paper as well as with the semantic weaknesses of the literature and therefore show how the syntactic grounding transforms their status within the deductive framework. Note of non-completeness of the literature: The following literature is the entire semantic body that exists. It is merely a snapshot of what there is.

11.1 HoTT as foundation: the formal context

The Homotopy Type Theory Book [1] provides the technical foundation for the formal layer of this paper. Its central innovation — the Univalence Axiom and the interpretation of identity types as paths — is presupposed throughout. The present work does not contribute to the development of HoTT itself; it *applies* HoTT to a philosophical deduction.

Voevodsky's original program [2] was motivated by the desire for machine-verifiable mathematics. Awodey [3] connected Univalence to mathematical structuralism, arguing that it formally captures the principle that structurally equivalent objects are identical. Tsementzis [4] developed this into a criterion for "structuralist foundations." Tsementzis and Halvorson [5] explored the broader philosophical implications of encoding mathematics in spatial terms ("points" and "paths").

Demarcation: These works treat HoTT as a foundation *for mathematics*. The present paper treats HoTT as a sufficient semantics for a deduction that is logically prior to mathematics — a deduction about the structure of possibility as such. The structuralism discussed by Awodey and Tsementzis concerns the identity of *mathematical objects*; the present paper concerns the identity of *anything that is*, insofar as it is structured. This is a different, and stronger, claim.

11.2 HoTT and philosophy: the interpretive context

Corfield [6] argues that modal HoTT provides a "new logic for philosophy," replacing first-order predicate logic as the formal language of choice. His work demonstrates applications to metaphysics (the distinction between objects and events), to the philosophy of language, and to geometry. It is the most ambitious existing attempt to bring HoTT into philosophy.

Demarcation: Corfield's project is programmatic: it shows that HoTT *can* be applied to philosophical problems. The present paper makes a specific deductive claim: that a single axiom, formalized in HoTT, *entails* an ethical imperative. Corfield provides the tools; the present paper uses them for a specific construction. Furthermore, the present paper claims that HoTT is sufficient but not necessary (Section 1.2) — a meta-theoretic claim that Corfield does not make.

11.3 Levinas: ethics and ontology

Levinas' *Totality and Infinity* [7] and *Otherwise than Being* [8] establish the philosophical framework in which the irreducibility of the Other is first articulated as a foundational principle. His insistence that ethics precedes ontology — that the face of the Other interrupts every totalizing system — is the central provocation to which Step 12 responds.

Subsequent scholarship has explored whether Levinas' position can be reconciled with ontology. Critchley [9] reads Levinas as demanding a "deconstruction" of ontology from within. Bernasconi and Critchley [10] collect approaches to the question. More recently, scholars working at the intersection of phenomenology and formal philosophy have asked whether Levinas' insights can survive formalization.

Demarcation: The present paper does not "apply" Levinas to type theory. It makes the stronger claim that Levinas identified a structural phenomenon — the irreducibility of alterity — but misdiagnosed its source. Step 12 derives the phenomenon *within* ontology, as a theorem about non-equivalent types under Univalence. The philosophical argument in Section 4 (the ontologization of Levinas) contends that Levinas' critique applies only to content-ontologies that mistake a semantic perspective for structure itself. A structural ontology that *deduces* alterity is immune to this critique — and performs what Levinas demands.

11.4 Process philosophy and Whitehead

Whitehead's *Process and Reality* [11] provides the deepest historical precedent for the claim that structure is fundamentally processual and relational. His "actual occasions" are constituted by their relations ("prehensions"), and his "creativity" functions as a principle of ontological productivity analogous to the axiom **A0**. Griffin [12] and Stengers [13] have developed process-philosophical frameworks in the continental and analytic traditions respectively.

Demarcation: The present deduction shares Whitehead's commitment to relational ontology but differs in method. Whitehead's system is speculative and non-formal; the present deduction is axiomatic and formally verifiable. What Whitehead articulated as a "categoreal scheme" — a set of interlocking philosophical categories — appears here as a chain of type-theoretic derivations from a single axiom. The formalization is not a translation of Whitehead; it is an independent construction that converges with his insights at specific points (especially: the priority of relation over substance, and the processual character of self-constitution).

11.5 The syntax-semantics distinction in philosophy of logic

The distinction between syntax and semantics, as deployed in this paper, has a long history in the philosophy of logic and language. Frege's *Begriffsschrift* [14] inaugurates the project of replacing semantic intuition with syntactic rigor. Wittgenstein's *Tractatus* [15] articulates the idea that logical structure "shows itself" rather than being said — an anticipation of the present paper's claim that syntax appears as semantics at higher h-levels. The later Wittgenstein [16] and the tradition of ordinary language philosophy (Austin [17]) shift focus to the performative and contextual dimensions of language, which the present paper recuperates in Section 9.

Derrida's *différance* [18] names the structural play of difference that precedes and enables meaning — a concept that resonates deeply with the type-theoretic derivation of difference from possibility (Step 1). However, Derrida refuses to ontologize *différance*, insisting that it is "neither a word nor a concept." The present paper, like Levinas but in the opposite direction, takes the Derridean insight and gives it ontological standing: difference *is* a structural consequence of the axiom, formally derivable.

Demarcation: None of these predecessors formalize the syntax-semantics distinction as a *deductive consequence* of a single axiom. The present paper does not merely *employ* the distinction; it *derives* the necessity of semantics from the formal properties of syntax (Step 9), and derives the necessity of the distinction itself from the incompleteness of self-reference (Step 11).

11.6 Formal ethics

Attempts to ground ethics in formal or logical structures include Kant's [19] derivation of the categorical imperative from the concept of rational agency, Gödel's unpublished ontological argument, and more recent work in deontic logic [20]. Badiou's [21] *Being and Event* uses set theory (specifically, Cohen's forcing) to formalize ontological claims, though not ethical ones.

Demarcation: The present paper differs from all of these in a specific way: it derives the ethical imperative not from a concept of agency, rationality, or duty, but from the *coherence conditions of self-referential structure*. Ethics is not a domain added to ontology; it is the operational consequence of ontological self-awareness. This is closer to Kant's spirit (ethics as a condition of rational coherence) than to deontic logic (ethics as a calculus of permissions and obligations), but it replaces Kant's appeal to "pure practical reason" with a type-theoretic derivation.

Acknowledgments

Draft. For discussion.

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